Seat No.	

W-966

Subject : Engineering Mathematics I   Code : 59177

F.E. SEM. – I and II (New Syllabus : Introduced from July 2013)

Day and Date : Wednesday, 04-06-2014   Time : 10 a.m. to 1.00 p.m.

Total Marks: 100

Instrunctions: 1) All the Quations are compulsory.
2) Figures to the right indicate full marks.
3) Use of non-programmable calculator is allowed.

SECTION -I

Q.1. Attempt any Three of the following **15**

a) Reduce the following matrix to the Echelon Form and hence determine the rank

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

b) Solve

$$2x_1 - x_2 + 3x_3 = 0$$
$$3x_1 + 2x_2 + x_3 = 0$$
$$x_1 - 4x_2 + 5x_3 = 0$$

c) Disscuss the consistency of following system of equations and if consistant solve

$$x_1 + x_2 + 2x_3 + x_4 = 5$$
$$2x_1 + 3x_2 - x_3 - 2x_4 = 2$$
$$4x_1 + 5x_2 + 3x_3 = 7$$

d) For what values of k the equations

$$x + y + z = 1,\ \ x + 2y + 4z = k,\ \ x + 4y + 10z = k^2$$

have a solution and solve them completely for one of the value of k.

Q. 2. Attempt any Three of the following **15**

a) Examine for linear dependence or linear independence of vectors
$(1 \quad 3 \quad 4 \quad 2), (3 \quad -5 \quad 2 \quad 6), (2 \quad -1 \quad 3 \quad 4)$
If dependent find the relation between them.

b) Find the Eigen values of the matrix

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$

and Eigen Vector corresponding to the smallest Eigen Value.

c) Verify Cayley-Hamilton's theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

d) Find the Eigen Values of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

and hence Eigen Values of A^{-1} *and adj.A*

Q 3 Attempt any Four of the following **20**

a) Find all the roots of $(1+i)^{1/5}$

b) Prove that $\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$

c) Use De Moivres Theorem to solve

$$x^8 + x^5 + x^3 + 1 = 0$$

d) If $\alpha + i\beta = \tanh\left(x + \frac{i\pi}{4}\right)$ then prove that $\alpha^2 + \beta^2 = 1$

e) Prove that $\tanh^{-1} x = \sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

SECTION-II

Q.4. Attempt ANY THREE from the following 15

a)Using Taylor's theorem express $(x-2)^4 - 3(x-2)^3 + 4(x-2)^2 + 5$ in powers of x.

b)Prove that

$\sec^2 x = 1 + x^2 + \frac{2}{3}x^4 + \ldots$.

c) If $\lim_{x\to 0} \frac{\sin 2x + p\sin x}{x^3}$ is finite then find the value of p and the limit

d) Show that $\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \ldots$

Q.5. Attempt ANY FOUR from the following 20

a) If $u = \log(x^3 + y^3 - x^2y - y^2x)$ Show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}$.

b) If $u = \tan^{-1}\left[\frac{x^3 + y^3}{x - y}\right]$. Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

c) If $x = v^2 + w^2,\ y = w^2 + u^2,\ z = u^2 + v^2$. show that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = 16uvw$

d)The resistance R of a circuit was found by using the formula $I = \frac{E}{R}$.If there is an error of 0.1 amperes in reading I and 0.5 volts in E. Find the corresponding possible percentage error in R, when reading are I=15 and E=100 volts.

e) Discuss the maximum and minimum of $x^3 + y^3 - 3axy$.

Q.6.Attempt ANY THREE from the following 15

a)Solve by Gauss elimination method

$$2x+y+z=10;\quad 3x+2y+3z=18;\quad x+4y+9z=16$$

b)Solve by Jacobi's iteration method, correct to two decimal places.

$$10x+y-z=11.19;\quad x+10y+z=28.08;\quad -x+y+10z=35.61$$

c) Solve the following system of equations by Gauss-Seidel method upto three iterations

$$10x+y+z=12;\quad 2x+10y+z=13;\quad x+y+5z=7$$

d) Determine the largest eigen value and the corresponding eigen vector of the matrix using power method $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$