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| Seat No. | |
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S.E. (Civil Engg.) (Part - II) (Semester - III)
Examination, April - 2016
ENGINEERING MATHEMATICS - III (Revised)
Sub. Code : 63338

Day and Date : Friday, 29 - 04 - 2016

Total Marks : 100

Time : 03.00 p.m. to 06.00 p.m.

- Instructions :
- 1) All questions are compulsory.
 - 2) Figures to the right indicate full marks.
 - 3) Use of Calculator is allowed.

SECTION - I

Q1) Solve any three of the following : **[18]**

a) $(D^2 - 6D + 13)y = e^{3x} \sin 4x + 3^x$

b) $(D^2 + 4)y = x \sin 3x$

c) $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$

d) The differential equation of strut of length l freely hinged at each end is

$$EI \frac{d^2y}{dx^2} + Py = \frac{-Wl^2}{8} \sin\left(\frac{\pi x}{l}\right). \text{ Find the bending of a strut at } x \text{ where}$$

$$\frac{P}{EI} = n^2.$$

Q2) Attempt any two of the following : **[16]**

- a) A vector field is given by $\vec{F} = (x^2 + xy^2)\mathbf{i} + (y^2 + x^2y)\mathbf{j}$. Show that \vec{F} is irrotational and find its scalar potential. Also find Grad (Div \vec{F}).

P.T.O.

- b) If \vec{a} is a constant vector and $\vec{r} = xi+yj+zk$ then prove that
- $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$
 - $\text{Div}(\vec{a} \times \vec{r}) = 0$
 - $\text{Div}(\vec{a} \cdot \vec{r})\vec{a} = a^2$
 - $\text{Curl}(\vec{a} \times \vec{r}) = 2\vec{a}$
- c) If $\vec{F} = (x + y + 1)i + j - (x + y)k$ then prove that $\vec{F} \cdot \text{Curl} \vec{F} = 0$ and if $\phi = x^3 + y^3 + z^3 - 3xyz$, find $\vec{r} \cdot \nabla \phi$ where $\vec{r} = xi+yj+zk$.

Q3) Attempt any two of the following : **[16]**

- a) From the following data find the line of regression of x on y and estimate x when y = 105

| | | | | | | | | | | |
|-----|----|-----|-----|-----|----|----|-----|----|-----|----|
| x = | 44 | 58 | 49 | 46 | 58 | 56 | 48 | 46 | 48 | 47 |
| y = | 88 | 114 | 102 | 113 | 91 | 89 | 102 | 93 | 114 | 94 |

- b) Fit a curve of the form $y = ab^x$ from the following data

| | | | | | |
|-----|-----|-------|-------|-------|-------|
| x = | 2 | 3 | 4 | 5 | 6 |
| y = | 144 | 172.8 | 207.4 | 248.8 | 298.5 |

- c) Estimate y when x = 12 by fitting a straight line to the following data

| | | | | | | | | |
|-----|------|------|------|------|------|------|-------|-------|
| x = | 1 | 2 | 4 | 5 | 6 | 8 | 9 | 10 |
| y = | 52.5 | 58.7 | 70.2 | 75.4 | 81.1 | 95.5 | 102.2 | 108.2 |

SECTION - II

Q4) Attempt any two of the following : **[16]**

- a) If ten percent of bolts produced by a machine are defective then determine the probability that out of 10 bolts, chosen at random
- one
 - none
 - at most 2
 - at least 2 bolts will be defective

- b) A skilled typist, on routine work, kept a record of mistakes made per day during 300 working days.

| | | | | | | | |
|------------------|-----|----|----|----|---|---|---|
| Mistakes per day | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| No. of days | 143 | 90 | 42 | 12 | 9 | 3 | 1 |

Fit a poisson distribution to the above data & hence calculate theoretical frequencies.

- c) i) The diameter of an electric cable is assumed to be continuous random variate with function $f(x) = 6x(1-x)$, $0 \leq x \leq 1$ verify that $f(x)$ is a probability density function.
- ii) Weights of 4000 students are found to be normally distributed with mean 50 kgs & standard deviation 5 kgs. Find the number of students with weights less than 45 kgs.

[Given : For S.N.V. Z area from $Z = 0$ to $Z = 1$ is 0.3413]

Q5) Attempt any three from the following :

[18]

- a) Find laplace transform of $t \cos 2t \cdot \cos t$.

- b) Find laplace transform of $\frac{d}{dt} \left(\frac{\sin t}{t} \right)$.

- c) Find inverse laplace transform of $\frac{s(s^2 + 2)}{(s + 1)^2(s^2 + 1)}$.

- d) Use laplace transform to solve

$$y'' - 4y' + 4y = 64 \sin 2t \quad y(0)=0, \quad y'(0)=1.$$

Q6) Attempt any two of the following :

- a) Show that the function $U=x^3-3xy^2$ is harmonic and find the corresponding analytic function.
- b) Evaluate using Cauchy's integral formula

i) $\int_c \frac{e^{2z}}{(z-1)(z-2)} dz$, where c is the circle, $|z|=3$

ii) $\int_c \frac{\cos \pi z}{z^2 - 1} dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$

c) Evaluate $\int_c \frac{z^2 - z + 1}{z - 1} dz$ where c is the circle

i) $|z|=1$

ii) $|z|=0.5$

